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The slow chopper and time-of-flight spectrometer in theory and experiment

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With 11 figures in the text

Introduction

As is well known our most copious source of free neutrons, the reactor, delivers a continuous spectrum of useful neutrons ranging from the MeV region down to subthermal energies. The problem to select a narrow energy interval out of this spectrum which can be successfully used in the investigation of the interaction of neutrons with matter has for a long time remained the main problem of the experimental neutron physics. Different instruments have been developed to meet the requirements. In the MeV region the problem is still more or less unsolved. In the low energy region from a few keV down to subthermal energies there exist three main types of instruments, choppers, crystal spectrometers and mechanical monochromators, which have been used with relative success to select narrow energy bands out of a reactor neutron spectrum. Here we will deal with the chopper-time-of-flight velocity selector, and, of the two types in common use, the fast and the slow chopper, we will concentrate on a theoretical and experimental study of a slow chopper spectrometer arrangement.

I. Principle of the chopper-time-of-flight spectrometry

The principle of the chopper-time-of flight spectrometer is well known. The chopper is in principle a fast rotating collimator, which, if placed in a collimated continuous neutron beam, transmits short bursts of neutrons. If such a neutron burst is allowed to travel a certain distance, neutrons of the different energies primarily present in the beam will be sorted out on the time scale: the faster neutrons will arrive at the end of the flight path earlier than the slower ones. If the flight time, t, for the given distance, l, is measured, the neutron velocity is given by $v = l/t$. The principle of the measurement is thus quite fundamental. The time measurement, which offers the main problem in this connection, must be performed by an electronic clock, the time analyzer, in such a way that the time measurement starts when the neutrons pass the chopper and stops when they reach the detector. Questions of vital importance which now arise are the following:

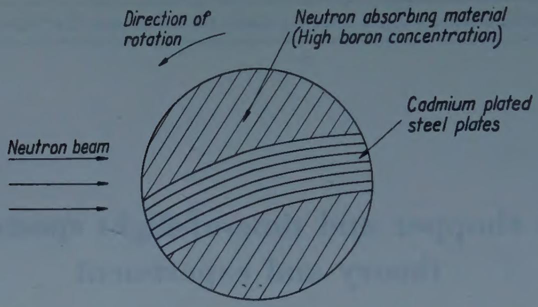
Direction of rotation
Neutron absorbing material (High boron concentration)
Cadmium plated steel plates
Neutron beam

Fig. 1. Cross section of slow chopper with curved slits.

1) What is the shape of the transmission curve of the chopper as a function of neutron wave length and time?
2) Does there exist a point in the chopper through which all neutron velocities present in the beam pass simultaneously, thus defining the zero of the time scale, i.e. the time when the analyzer should be started and from which point the flight paths should be measured?
3) What is the energy resolution of the chopper-time-of-flight spectrometer?

To define the problem we shall first assume that a parallel neutron beam impinges upon a cylindrically shaped chopper of radius r through which are cut curved slits of width $2d$ and which have a radius of curvature R (fig. 1). The chopper rotates with an angular velocity ω. We shall first derive the answer to the first question regarding the transmission. The chopper is of so-called Fermi-type.

II. The chopper transmission as a function of neutron wave length

We shall first investigate the shape of a neutron path in a rotating system. If it is assumed that the neutron at the time $t=0$ intersects the x-axis in a fixed (laboratory) system, and in this system moves along the line $x=a$ with velocity v, it is easily proved that in a rotating system x', y' connected to the chopper and with the z' axis coinciding with the axis of rotation, the neutron path is a parabola, if we consider only small angles of rotation ($\theta=\omega t$ small):

$$y'^2 = \frac{(v-a\,\omega)^2}{\omega\left(v-\dfrac{a\,\omega}{2}\right)}\,(x'-a) \tag{1}$$

The radius of curvature, R, at the vertex is

$$R=\frac{1}{2}\frac{v}{\omega}-\frac{3}{4}a+\ldots, \tag{2}$$

where it is assumed that $a \ll v/\omega$. This assumption, which is always fulfilled for realistic choppers, means, that we can without loss of generality confine our discussion to neutron paths for which $a \cong 0$. The relation (2) merely expresses the fact that the neutron paths connected with different small values of a, almost have a common centre of curvature, a fact which is commonly used in the construction of choppers with curved slits. For $a = 0$ and the neutron velocity v_0 the equation of the neutron path is $y'^2 = v_0/(\omega)x'$ and for a different velocity v the neutron path is described by $y'^2 = v/(\omega)x'$. If we now consider a system connected to the chopper in which the path of the neutrons of velocity v_0 is a straight line we find that the equation for the path of neutrons of velocity v is given by

$$y'^2 = \frac{1}{\frac{\omega}{v} - \frac{\omega}{v_0}}\, x'' \tag{3}$$

where $x'' = x' - x_0' = y'^2\omega/v - y'^2\omega/v_0$. We have now referred the path of neutrons of velocity v to that of neutrons of velocity v_0. Even in this system the neutron path is a parabola with a radius of curvature at the vertex given by

$$R = \frac{1}{2\omega \left| \frac{1}{v} - \frac{1}{v_0} \right|} \tag{4}$$

This radius of curvature is in practice quite large, of the order of 1 meter, while the choppers usually are of small dimensions with radius of the order of 3–6 cm. It is thus a very good approximation to replace the parabolas (eq. 3) with circles. If we now consider a particular slit of width $2d$, assumed to be curved with a radius of curvature of $R_0 = v_0/(2\omega)$, we find that the path of a neutron with velocity $v \neq v_0$ is a circle as illustrated in figure 2. The transmission through the slit is obviously a

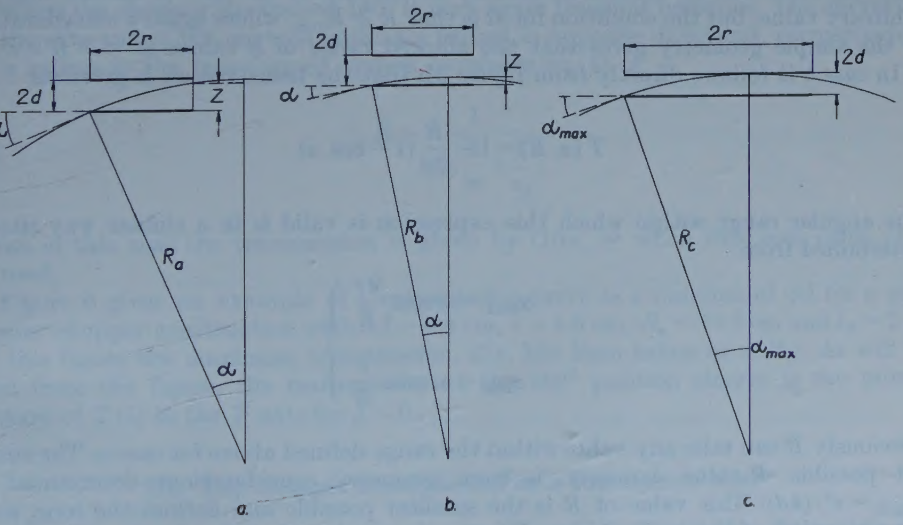

Fig. 2. Possible neutron paths through the chopper. The curved slit is transformed into a straight one. Neutron paths approximately circular arcs.

function of the angle α of incidence and of the relative radius of curvature R as given by equation 4. The transmission is given by

$$T(\alpha, R) = \frac{2d - z}{2d} \tag{5}$$

where z is a function of α and R. To calculate the total transmission for a constant R the eq. 5 has to be integrated over the possible angles, α, of incidence. In order to calculate $z(\alpha, R)$ the possible range of α-values has to be divided into two regions as illustrated by figure 2 a & b:

a. The circle of radius R has its maximum outside the slit.
b. The circle has its maximum within the slit.

From simple geometry it follows that in *case a*, the transmission is given by

$$T(\alpha, R) = 1 - \frac{R}{2d}\left\{\sqrt{1 - \left(\sin\alpha - \frac{2r}{R}\right)^2} - \cos\alpha\right\} \tag{6}$$

The angular range within which this expression is valid is determined from

$$\left.\begin{aligned} \alpha_{\max} &\cong \arcsin\left\{\frac{d}{r} + \frac{r}{R} - \frac{1}{2}\left(\left(\frac{d}{r}\right)^3 + \frac{rd}{R^2}\right)\right\} \\ \alpha_{\min} &= \arcsin\frac{2r}{R} \end{aligned}\right\} \tag{7}$$

Here $\alpha_{\max}$ follows from the condition $z = 2d$, and $\alpha_{\min}$ follows from the condition that the circle has its maximum exactly at the end of the slit. R cannot take an arbitrary value, but the condition for R is that $R \geqslant R_{\min}$, where again a consideration of the simple geometry gives that the allowed range of R values is $\infty \geqslant R \geqslant r^2/d$.

In *case b* it follows directly from figure 2b that the transmission is given as

$$T(\alpha, R) = 1 - \frac{R}{2d}(1 - \cos\alpha) \tag{8}$$

The angular range within which this expression is valid is in a similar way simple determined from

$$\left.\begin{aligned} \alpha_{\max} &= \arcsin\frac{2r}{R} \\ \alpha_{\min} &= \arcsin\frac{r}{R} \end{aligned}\right\} \tag{9}$$

Obviously R can take any value within the range defined above for case a. The smallest possible R-value, however, is from geometry considerations determined by $R_{\min} = r^2/(4d)$. This value of R is the smallest possible and defines the long wave length cut off of the transmission. To sum up we thus find that in the wave length region determined by $\infty \geqslant R \geqslant r^2/d$ the total transmission is given by

$$T(R)=2\int(\text{eq. }6)\,d\alpha+2\int(\text{eq. }8)\,d\alpha$$

with the angular ranges given by (7) and (9) respectively. The factor 2 takes into account, that the transmission expressed in the angle α is symmetric.

In the wave length region determined by $r^2/d \geqslant R \geqslant r^2/(4d)$ the transmission is described by eq. 8 with the following limits of integration:

$$\alpha_{\max}=\arcsin 2\sqrt{\frac{d}{R}-\left(\frac{d}{R}\right)^2}\quad\text{and}\quad \alpha_{\min}=\arcsin\frac{r}{R}$$

The integrations may be performed after a number of series expansions with the result

$$\boxed{T(R)\cong\begin{cases}\dfrac{d}{r}\left\{1-\dfrac{1}{6}\dfrac{r^4}{d^2R^2}\right\}, & \infty \geqslant R \geqslant \dfrac{r^2}{d} \qquad (10\text{ a})\\[2ex] \dfrac{8}{3}\sqrt{\dfrac{d}{R}}-\dfrac{2r}{R}+\dfrac{1}{6}\dfrac{r^3}{dR^2}, & \dfrac{r^2}{d}\geqslant R\geqslant\dfrac{r^2}{4d} \qquad (10\text{ b})\end{cases}}$$

Terms which have been neglected, are of the order of $(d/r)^3$ or less. As for actual choppers d is of the order mm and r of the order cm, the terms neglected are certainly $< 1\,\%$. If we now remember the definition (4) of R we find that

$$\frac{1}{R}=\frac{2m\omega}{h}\,|\lambda-\lambda_0|.$$

In a region defined by (10a) the transmission is thus parabolic in the variable $|\lambda-\lambda_0|$. The maximum transmission occurs for $\lambda=\lambda_0$ and is then given by d/r.

When the chopper has turned 180° it may again transmit neutrons. The curvature of the slits and of the neutron paths are turned in opposite directions, corresponding to a radius in the transformed system (compare eq. 4) of

$$R=\frac{1}{2\omega\left|\dfrac{1}{v}+\dfrac{1}{v_0}\right|}$$

Even in this case the transmission is given by (10a, b) when this new value of R is used.

Figure 8 gives an example of a transmission curve as a function of $\omega\lambda$ for a particular chopper construction with $2d=0.4$ cm, $r=4.8$ cm, $R_0=74.5$ cm and $\lambda_0=2$ Å. In this figure the maximum transmission, d/r, has been taken as unity. As will be seen from the figure, the transmission at the 180° position simply is the mirror picture of $T(\lambda)$ in the T axis for $\lambda<0$.

III. The chopper transmission as a function of time

We shall now turn to question number 2 of section I. The study of the time dependence of the transmission, $T(t,R)$, may simply be reduced to a study of the transmission as a function of the angle, α, of incidence, or—what amounts to the same thing—as a function of the angle of rotation of the chopper. The relations (6) and (8) derived in the preceding section, which just give $T(\alpha, R)$, may be written after series expansion with due regard to the fact that the angles α are small:

$$T(\alpha, R) \cong \begin{cases} 1 + \dfrac{r^2}{d \cdot R} - \dfrac{r}{d}\alpha, & \dfrac{d}{r} + \dfrac{r}{R} \geqslant \alpha \geqslant \dfrac{2r}{R} \\ 1 - \dfrac{R}{4d}\alpha^2, & \dfrac{2r}{R} \geqslant \alpha \geqslant \dfrac{r}{R} \end{cases} \tag{11}$$

These expressions are valid if $\infty \geqslant R \geqslant r^2/d$. In an analogous manner the transmission in the region $r^2/d \geqslant R \geqslant r^2/(4d)$ is given by

$$T(\alpha, R) \cong 1 - \frac{R}{4d}\alpha^2, \quad 2\sqrt{\frac{d}{R}} \geqslant \alpha \geqslant \frac{r}{R} \tag{12}$$

It is to be emphasized that the transmission function $T(\alpha, R)$ is symmetric around the angle $\alpha = r/R$ and that (11) and (12) above give the transmission measured at the entrance side of the chopper, where the angle $\alpha = 0$ corresponds to the transmission maximum for neutrons of velocity $v = v_0$.

Let us now investigate in detail the transmission as a function of time. It is sufficient to discuss just *those neutrons which correspond to the peak of the transmission functions* (11) and (12) above. We consider only a parallel incident neutron flux. As is easily seen, neutrons of velocity $v < v_0$ enter the chopper at an angle $\alpha = r/R$ *before* neutrons of velocity $v = v_0$, which enter the chopper at an angle $\alpha = 0$ or a time $t = 0$. Similarly, neutrons of velocity $v > v_0$ enter the chopper at an angle $\alpha = r/R$ *after* neutrons of velocity $v = v_0$. The difference in flight time necessary to reach the centre of the slit for neutrons of velocity v and v_0 respectively, is obviously

$$|\delta t| = r \left| \frac{1}{v} - \frac{1}{v_0} \right|$$

During this time interval the chopper turns an angle

$$\omega \delta t = r\omega \left| \frac{1}{v} - \frac{1}{v_0} \right|$$

But this angle is exactly half of the difference in angle of incidence,

$$\alpha - 0 = \frac{r}{R} = 2\omega r \left| \frac{1}{v} - \frac{1}{v_0} \right|$$

in our relative system, which means, that neutrons of velocity v and v_0 that enter the slit an angle r/R before neutrons of velocity v_0 reach the centre of the slit simultaneously with neutrons of velocity v_0. This follows from the fact that angles of rotation of the chopper are in general exactly half the change in angle of the corresponding tangents to the neutron path through the slit as seen in our relative system. The important conclusion is thus that the centre of the chopper is "the

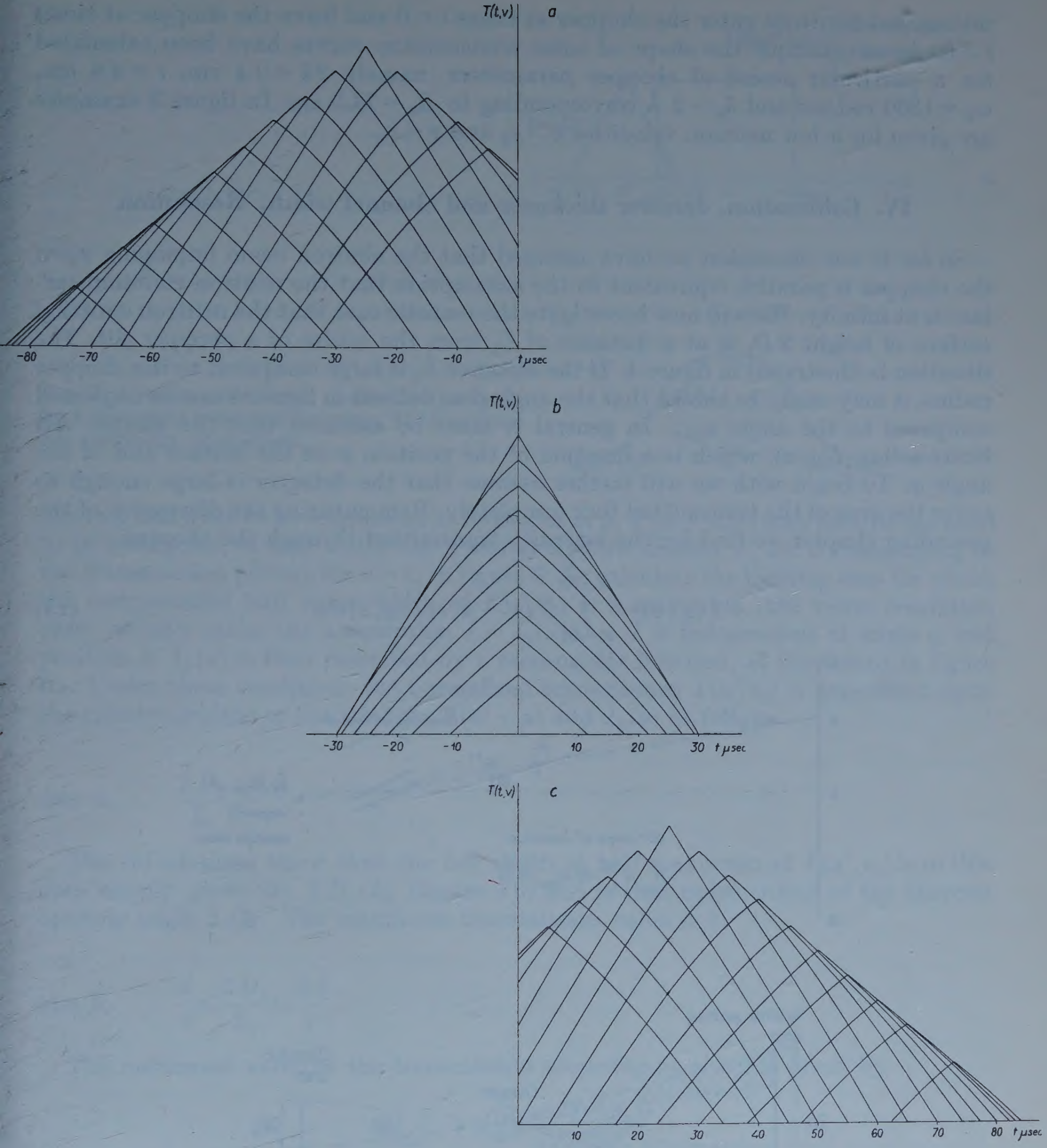

Fig. 3. Calculated transmission curves as a function of time.

chopping point" from which flight paths should be measured. The measurement of flight times should be started when the chopper is in its symmetry position such that the neutron paths forming straight lines in the laboratory system are tangent to the centre of the curved slit.

The complete picture of transmission is now clear. If time "0" is selected as that time when the transmission maximum, $T(\alpha, R)_{\max}$, passes the centre of the slit, this means that neutrons enter the chopper at times $t < 0$ and leave the chopper at times $t > 0$. As an example the shape of some transmission curves have been calculated for a particular choice of chopper parameters, namely $2d = 0.4$ cm, $r = 4.8$ cm, $\omega_0 = 1330$ rad/sec and $\lambda_0 = 2$ Å corresponding to $R_0 = 74.5$ cm. In figure 3 examples are given for a few neutron velocities $v > v_0$ and $v < v_0$.

IV. Collimation, detector thickness and channel width. Resolution

So far in our discussion we have assumed that the neutron beam impinging upon the chopper is parallel, equivalent to the assumption that the neutron emitting surface is at infinity. We will now investigate the realistic case that the neutron emitting surface of height $2D_1$ is at a distance of L_1 from the centre of a chopper slit. The situation is illustrated in figure 4. If the distance L_1 is large compared to the chopper radius, it may easily be shown that the angle β as defined in figure 4 can be neglected compared to the angle $\alpha_{\max}$. In general it must be assumed that the surface AB emits a flux $I(\varphi, x)$, which is a function of the position x on the surface and of the angle φ. To begin with we will further assume that the detector is large enough to cover the area of the transmitted flux completely. Remembering the discussion of the preceding chapter we find for the intensity transmitted through the chopper

$$I(\alpha', v) = \int_{\alpha' - d/r}^{\alpha' + d/r} T(\alpha' - \alpha, v) I(\varphi, x)\, d\alpha \tag{13}$$

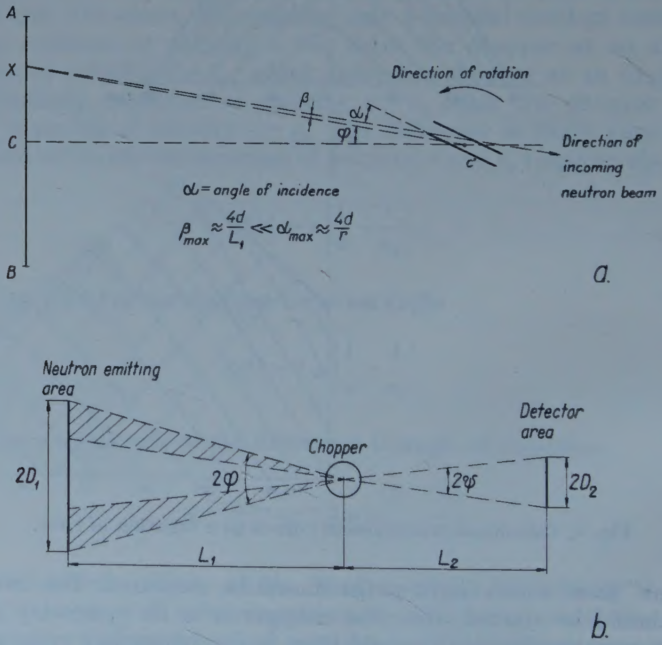

Fig. 4. Principle of spectrometer geometry.
a. Chopper slit sweeping over the beam area. Beam collimation determined only by the dimensions of the neutron emitting area $\overline{AB} = 2D_1$, the distance $\overline{CC'} = L_1$ and chopper slit width $2d$.
b. No neutrons can reach the detector from shaded areas if $2\psi < 2\varphi$.

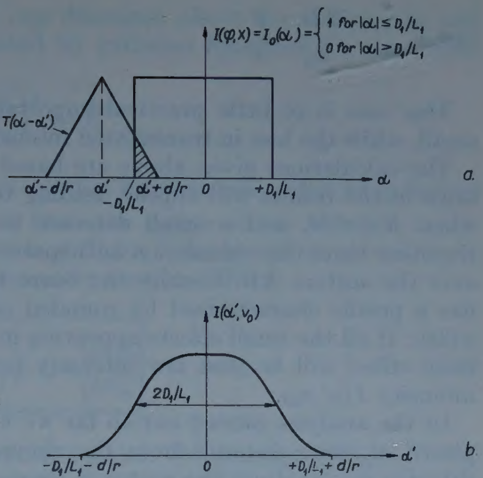

Fig. 5. A rectangular beam profile $I(\alpha')$ as seen by a triangular resolution function. The functions are arbitrarily normalized.

where φ and x are to be considered as functions of α. $T(\alpha' - \alpha, v)$ may here be assumed to be triangular, of base width $2d/r$, and symmetric around α', i.e. corresponding to the transmission picture for $v = v_0$ in figure 3. To calculate the limiting case for which the compounded half value width of $I(\alpha', v_0)$ is a maximum (the worst resolution case) we now make the assumption $I = I_0(\alpha)$, i.e. I is independent of angle φ and position x. $I_0(\alpha)$ is then described by a rectangular function, as illustrated in figure 5a. Under these conditions the normalized transmission $I(\alpha', v_0)$ is dependent upon the relative widths of the triangle $T(\alpha' - \alpha)$ and $I_0(\alpha)$ as follows:

case a, $\qquad \dfrac{2D_1}{L_1} \geqslant \dfrac{2d}{r}$

The calculations show that the full width at half maximum of $I(\alpha', v_0)$ is in this case simply given by $2D_1/L_1$ (figure 5 b) and is thus independent of the chopper opening angle $2d/r$. The maximum transmission value is 1.

case b, $\qquad \dfrac{d}{r} \leqslant \dfrac{2D_1}{L_1} \leqslant \dfrac{2d}{r}$

The maximum value of the transmission occurring at $\alpha' = 0$ is given by

$$I_{\max} = 1 - \frac{(d/r - D_1/L_1)^2}{2(d/r)^2}$$

and is thus $\leqslant 1$. As is easily shown, *the full width of* $I(\alpha', v_0)$ *at half maximum* is

$$\frac{2D_1}{L_1} + \frac{2d}{r} - 2\sqrt{\left(\frac{d}{r}\right)^2 - \left(\frac{d}{r} - \frac{D_1}{L_1}\right)^2} \tag{14}$$

i.e. it varies from $\dfrac{2d}{r}$ when $\dfrac{D_1}{L_1} = \dfrac{d}{r}$ to $(3 - \sqrt{3})\,\dfrac{d}{r}$ when $\dfrac{2D_1}{L_1} = \dfrac{d}{r}$

case c, $\dfrac{2D_1}{L_1} \leqslant \dfrac{d}{r}$

This case is of little practical importance as the gain in the halfwidth value is small, while the loss in transmitted intensity is large, from 25 to 100 %.

The calculations given above are based on the case $v = v_0$. If $v \neq v_0$ minor deviations in the results will appear, leading to a small increase in the half-width value when $R \geqslant r^2/d$, and a small decrease in this value when $r^2/d \geqslant R \geqslant r^2/(4d)$. On the other hand this calculation anticipates that the neutron emission rate is constant over the surface AB. Usually the beam from a collimator or a reactor beam tube has a profile characterized by rounded corners, which tends to decrease the half-width. If all the small effects appearing in practice are taken into consideration, the main effect will be that low intensity tails are added to the transmitted neutron intensity $I(\alpha', v_0)$.

In the analysis carried out so far we have explicitly assumed that the detector, placed at some distance from the chopper, has large dimensions. If instead, the detector as seen from the centre of chopper subtends an angle $2\psi \cong 2D_2/L_2$ subject to the condition that $2\psi < 2D_1/L_1$—as illustrated schematically in figure 4b—it is obviously the angular opening of the chopper and the detector angle, which together determine the angular width of the chopper burst. The discussion carried through above then remains valid if D_1/L_1 is replaced by D_2/L_2. Furthermore, if this is the case, the assumption of a constant neutron flux over the neutron emitting area now has to be replaced by the condition: a constant detection efficiency over the detector area, which in practical applications is far more realistic.

If we now want to express the results on a time scale, we may do so by just multiplying by $1/\omega$. In conclusion, we find that the full widths at half maximum of the transmitted chopper bursts are:

$$\boxed{\begin{array}{lll} \text{a.} & \delta t_\omega = \dfrac{2D}{\omega L} & \text{if} \quad \dfrac{2D}{L} \geqslant \dfrac{2d}{r} \\ \text{b.} & \dfrac{2d}{\omega r} \geqslant \delta t_\omega \geqslant (3-\sqrt{3})\dfrac{d}{\omega r} & \text{if} \quad \dfrac{2d}{r} \geqslant \dfrac{2D}{L} \geqslant \dfrac{d}{r} \end{array}} \qquad (15)$$

Here D and L should be referred to the neutron emitting surface if $D_1/L_1 < D_2/L_2$ or to the detector geometry if $D_2/L_2 < D_1/L_1$. The index ω on δt indicates that the half width values are dependent upon the chopper's "sweep velocity", ω.

We now have considered that part of the total resolution of the chopper-time-of-flight spectrometer which originates from the chopper geometry and collimation properties of the system. In practice two more factors generally contribute to the final time resolution, namely the detector thickness and the electronic channel width of the time analyzer. If the detector has an effective thickness d_1, the flight time through the detector for neutrons of velocity v is $\delta t_d = d_1/v$. Nothing else can be said generally about this uncertainty in neutron flight time, as this time error depends upon the particular detector construction used. If the detector is a thick BF_3 counter, account has to be taken to the self-screening effect. Similarly the electronic channel width, δt_c, may vary in shape, but usually it is rectangular. These two flight-time uncertainties, which are both independent of the chopper speed of rotation might be added in a way similar to the one discussed above for collimation and chopper geometry. If they are approximated by gaussian functions we may write

$$\delta t_e = \sqrt{\left(\frac{d_1}{v}\right)^2 + \delta t_c^2}$$

Finally, if all four contributions to the resolution function are added as gaussians we find $\delta t = \sqrt{\delta t_\omega^2 + \delta t_e^2}$. If the flight path from the centre of the chopper to the effective centre of the detector is l, the time-of-flight, t, for neutrons of velocity v is l/v, and the total resolution of the spectrometer may be written

$$\boxed{\frac{\delta t}{t} \cong c \cdot \frac{d\,v}{r\,\omega\,l} \sqrt{1 + \frac{(\delta t)_e^2}{(\delta t)_\omega^2}}} \tag{16}$$

The ideal value of c falls in the region between 2 and $3 - \sqrt{3}$ corresponding to (15b), as this geometry gives a maximum transmission between 100 and 75%. We may in a practical case make such a choice that $c \cong 1.4$. It is then ideal to use such a detector and to choose such a channel width that $\delta t_e = \delta t_\omega$, in which case the resolution comes out as

$$\frac{\delta t}{t} \cong \frac{2\,d\,v}{r\,\omega\,l} \tag{16'}$$

This choice of parameters corresponds to the optimum working conditions of the chopper-time-of-flight spectrometer, representing maximum transmission and highest resolution simultaneously.

V. Choice of chopper parameters

Faced with the practical problem to construct a chopper-time-of-flight spectrometer for use in a particular field, one has to make the proper choice of a series of parameters: the chopper radius r, the slit width $2d$, the angular velocity ω, the length of the flight path l, and the resolution $\delta t/t$ measured at a neutron wave length λ_0. The choice of resolution determines, as we have seen in the preceding section, a series of other parameters such as collimation, detector and channel width. If we here concentrate on the central part of the apparatus, the chopper, we thus have six parameters to be determined. With the aid of the preceding sections and a few new conditions we may reduce the number of degrees of freedom considerably.

The first reasonable requirement to be put on the spectrometer is that all the neutrons from a particular neutron burst should have reached the detector before the chopper opens up again, i.e. no overlap is allowed. But this means that the slowest neutron transmitted, velocity $v_{\min}$, should run the flight path, l, in a time $\leqslant 2\pi/\omega$, which is the time between two consecutive openings of the chopper:

$$\frac{2\pi}{\omega} v_{\min} = l \tag{17}$$

But $v_{\min}$ is determined from

$$R_{\min} = \frac{1}{2w\left(\frac{1}{v_{\min}} - \frac{1}{v_0}\right)} = \frac{r^2}{4d} \tag{18}$$

If we now assume that the optimum resolution condition given by eq. 16′ is required, we may combine 16′, 17 and 18 to give:

$$\boxed{\frac{d}{r} = \frac{\pi \cdot \left(\frac{\delta t}{t}\right)}{\frac{l}{r}\left(\frac{\delta t}{t}\right) + 1}} \tag{19}$$

where the resolution $\delta t/t$ is taken at $v = v_0$. For constructional reasons it is natural to predetermine ω, and further if $\delta t/t$ is chosen to have a definite value at the particular neutron velocity v_0, one has the solution given by (16′)

$$\boxed{\frac{d}{rl} = \frac{\omega}{2v_0} \cdot \left(\frac{\delta t}{t}\right)} \tag{20}$$

A further condition might be applied if for instance the chopper is desired to give a fairly flat transmission in a region of neutron velocities round v_0, corresponding to the parabolic part of the transmission curve (10a). Eq. 4 gives

$$\boxed{\frac{r^2}{d} = \frac{h}{2mw|\lambda_c - \lambda_0|}} \tag{21}$$

If $|\lambda_c - \lambda_0|$ is predetermined, the simultaneous solution of equations 19, 20 and 21 uniquely determines d, r and l.

VI. Experimental study of a time-of-flight spectrometer

a. Construction of the spectrometer

The construction of the chopper-time-of-flight spectrometer in operation at the Stockholm reactor, $R1$, has already been described in various articles [1, 2, 3]. The principles of the arrangement are given in figure 6. The chopper parameters have been chosen such that the transmission should have a fairly flat region round 2 Å at a speed of rotation of 12,700 r.p.m. There are ten slits of width 0.4 cm and the effective radius of the chopper is 4.8 cm. With a resolution of about 4 % at 2 Å the flight path is of the order 3 meters. The angular opening, 2ψ, of the detector as seen from the chopper is $2D_2/L_2 \cong 20/300$, i.e. about 3.8°. The detector, made up of 29 $B^{10}F_3$ counters arranged in three layers, has a geometric thickness of 8.2 cm and —if the boundary effects are taken into consideration—an effective thickness of $\sim$6.5 cm. For further description see references given above.

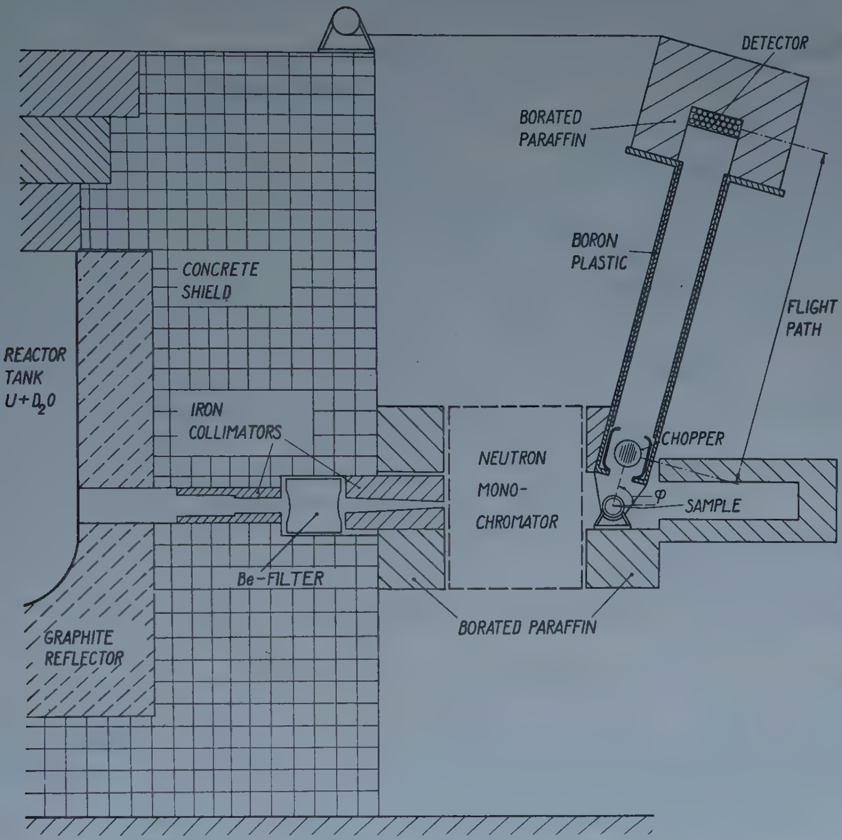

Fig. 6. The principles of the construction of the time-of-flight spectrometer. The velocity selection of the incident beam might be performed either by the beryllium filter or by a device such as a crystal monochromator, a mechanical monochromator etc, or by a combination of both.

b. Experimental determination of the transmission curve

Experimentally the transmission curve may be determined in different ways. A simple method is to study how a known neutron spectrum is deformed after transmission through the chopper. If the intensity distribution in the spectrum before transmission is described by $I_0(\lambda)$, the desired transmission function follows from $T(\lambda) = I(\lambda)/(I_0(\lambda))$ for a constant speed of rotation of the chopper. As the general transmission curve is best described as a function of $\omega\lambda$, one can also proceed in such a way that a fairly flat spectrum is studied as a function of ω. The intensities transmitted at one or several selected wave lengths are then studied as a function of ω. If the spectrum used for the study does not cover a very wide wave length region this is the only way to perform the measurement. For each ω a range of $\omega\lambda$ values is then covered. This method was used in the present experiment. With the spectrometer arm in the 0° position the spectrum transmitted through a 25 cm thick beryllium filter cooled to liquid nitrogen temperature was studied at a

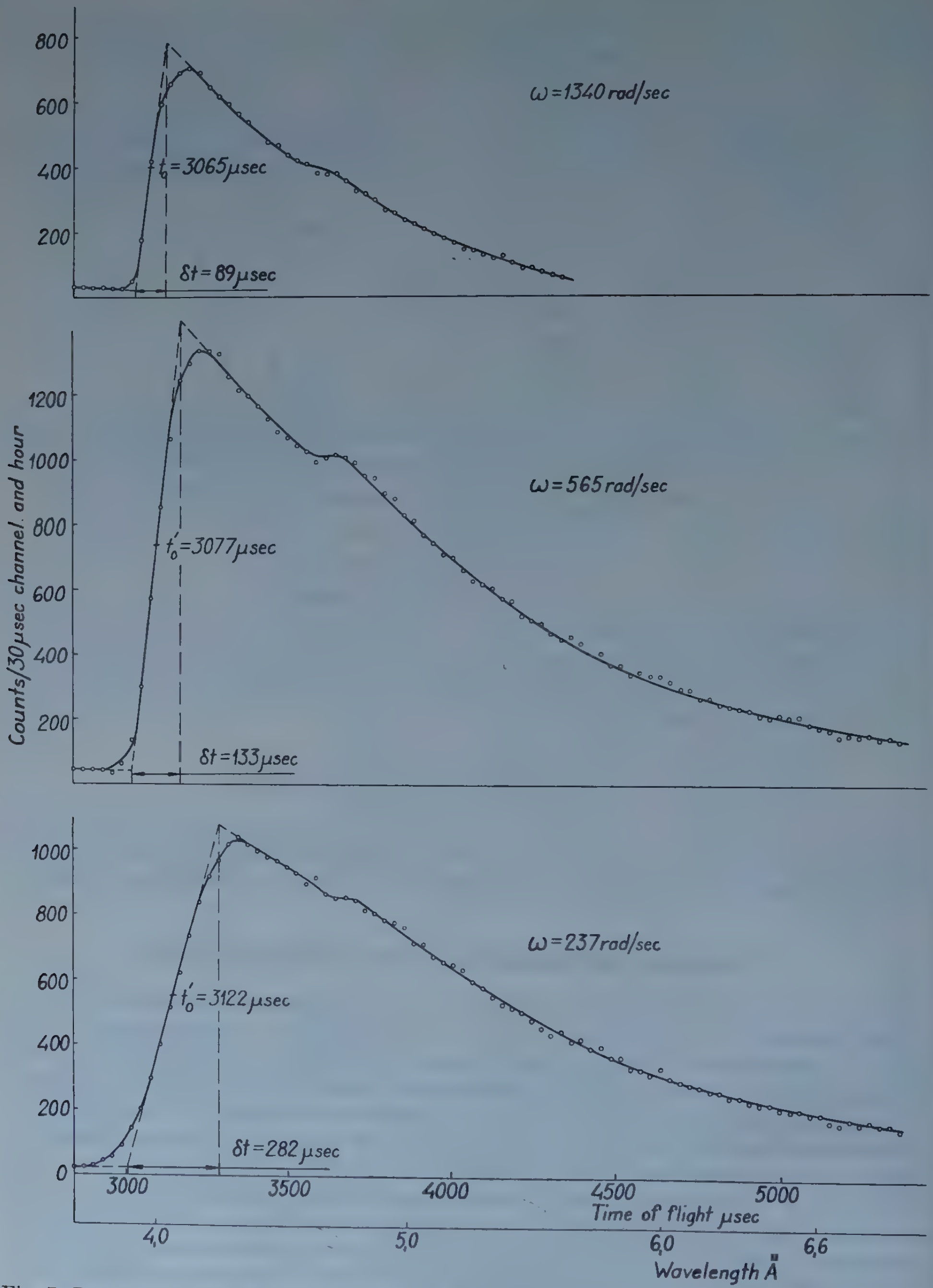

Fig. 7. Beryllium filtered reactor spectrum as observed at different chopper speeds. Three facts are observed: a. the intensity distributions changes; b. the slope of the beryllium break increases with decreasing ω; c. the position of the break moves towards longer flight-times as the chopper speed is decreased.

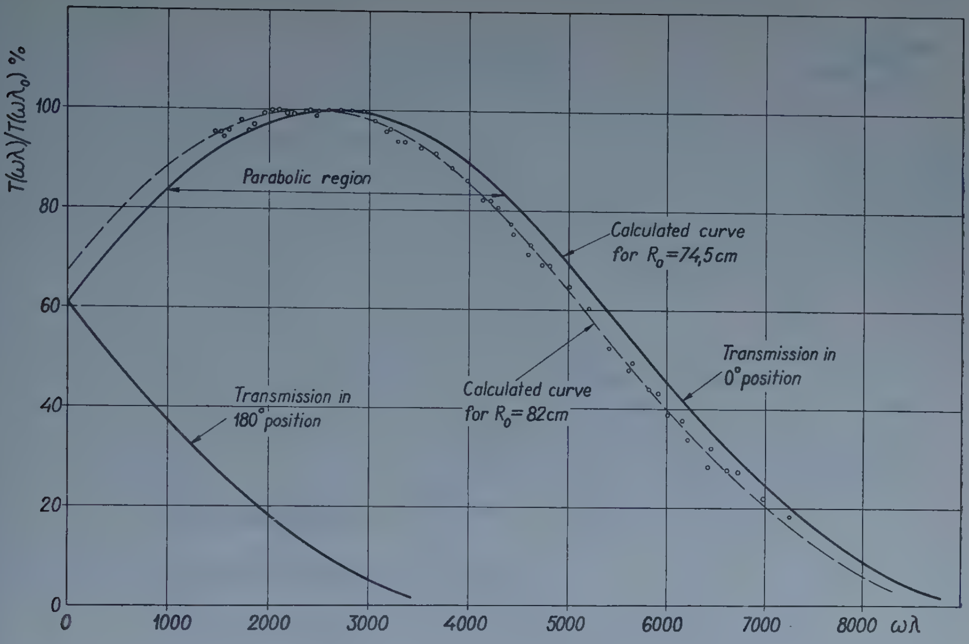

Fig. 8. Observed and calculated transmission curves.

series of speeds of rotation from 13,000 to 2250 r.p.m. Intensities transmitted in a wave length region from 4.2 to 6.6 Å were used to calculate the transmission curve. (See figure 7.) For any particular wave length in this region the transmitted intensity was plotted as a function of $\omega\lambda$, and the resulting curve was normalized to its maximum value. The resulting experimentally determined transmission function $T(\omega\lambda)$ is given in figure 8 together with the calculated one. In the calculation it was assumed that the mean radius of curvature of the slits was 74.5 cm in accordance with construction. As will be seen from figure 8 the experimentally determined curve is displaced some 10 % towards lower $\omega\lambda$ values compared to the calculated one, which should correspond to a radius of curvature of the slits of 82 cm and a maximum transmission at 1.8 Å instead of the calculated 2 Å at $\omega = 1330$ rad/sec. It is to be observed that the shape of the experimental curve very well agrees with the calculated, as is seen if this later curve is displaced 10 % towards lower $\omega\lambda$ values. This might indicate, that a weakening in the material has occurred after 18 months of use at high speed, such that the slits have become slightly straightened out.

Our chopper is designed for slow neutrons and is constructed with cadmium as neutron absorbing material. Consequently, it should not be operated in a flux containing a high percentage of resonance neutrons, as these will pass right through the set of slits as if the chopper contained just one large opening. To demonstrate this the reactor beam hole used in this experiment was covered with a cadmium plate of 0.5 mm thickness, which effectively removes the beryllium filtered spectrum and any background thermal neutrons. The only flux present will then be just the resonance and fast neutron background. The resulting background curve is shown in figure 9. Two broad peaks are observed, one corresponding to the chopper 0°

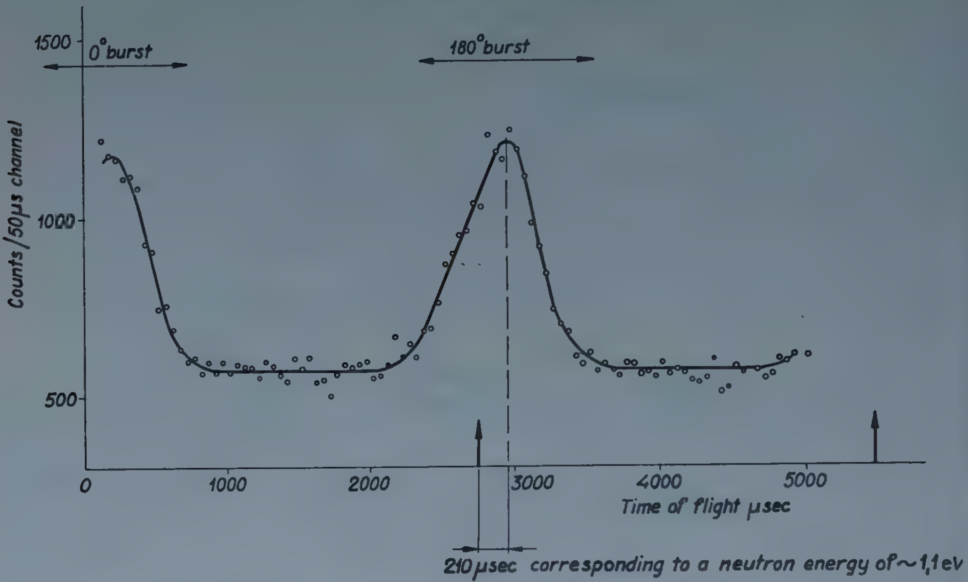

Fig. 9. Observed background spectrum of resonance neutrons.

position and the other to the 180° position. The flight time of 210 μsec measured from the 180° peak allows one to estimate the average energy of these background neutrons to be about 1–2 eV. Further the angular width of the observed 180° peak rather well agrees with the full angular opening of the chopper if all the slits were absent.

c. *Experimental determination of the resolution of the instrument*

As has been described in detail in earlier publications [2, 3], the part of the reactor neutron spectrum transmitted through a beryllium filter is well suited for a determination of the half-width of the resolution function. The sharp Bragg cut-off at 3.952 Å at liquid nitrogen temperature will be rounded off when a resolution function of finite width is applied, so that its slope will be finite. The projection on the time axis of the tangent at the point of inflexion of the sloping cut-off will give an approximate value of the full width at half maximum, δt (actually 1.06 δt if the resolution function is supposed to be of gaussian shape). As was shown in section IV, the total resolution function is generally composed of four contributions, two of which, namely the width of the chopper pulse and the sweep time over the detector, are dependent on ω, and two others, the flight time through the detector and the channel width, which are not. Thus if ω is varied and the width of the beryllium break is studied as a function of ω, the resolution width may be written: $\delta t = 1/\omega\sqrt{a^2 + b^2\omega^2}$, where a represents the combined angular half-width of the chopper opening and collimation angle and $b = \delta t_e$ (compare eqs. 14 and 16). Experimentally, the same runs which were made to study the transmission curve were also used to analyze the width of the beryllium break. As in the case of the transmission curve, these data were corrected for the

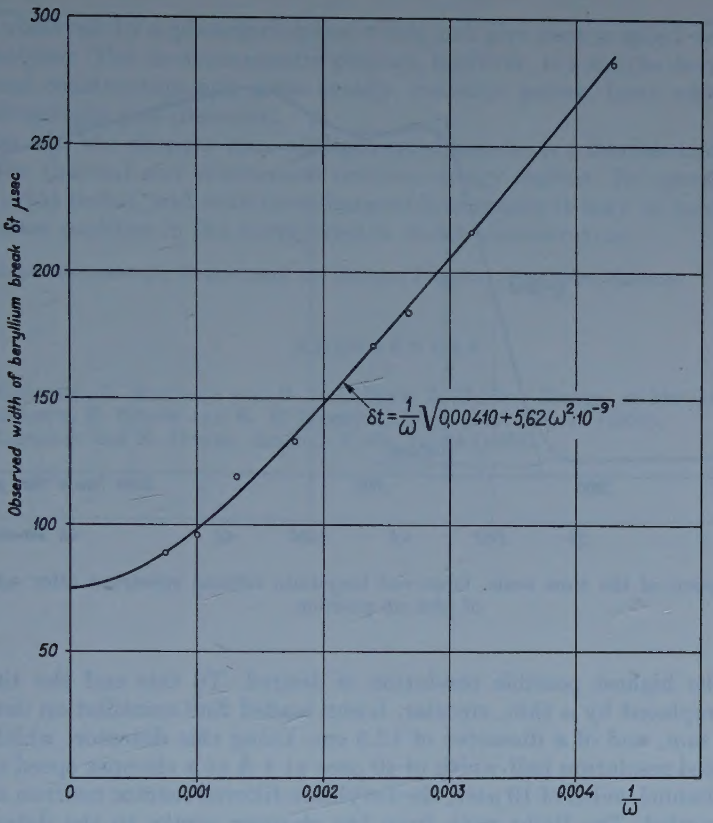

Fig. 10. Observed width of beryllium break as a function of $1/\omega$. The solid line is the theoretically calculated width given by the function $\delta t = 1/\omega \sqrt{a^2 + b^2\omega^2}$.

dead time of the analyzer, air absorption, self screening of the detector and scattering in the 25 mm thick layer of aluminium which the neutrons had to pass through [1].

The monotonously increasing width of the beryllium break with decreasing speed of rotation is illustrated in figure 7. Figure 10 shows the experimentally determined resolution width as well as the theoretically calculated one. As seen from the figure the agreement between theory and experiment is satisfactory.

d. Calibration of the time-of-flight scale

The calibration of the time scale of the spectrometer is simply performed in the way that neutrons of a well known wave length are sent through the chopper. The flight time, t, for the distance, l, is simply $t = ml/(h)\lambda$, where m = the neutron mass and h = Plancks constant. When the calibration is correct the measured flight time should agree exactly with the calculated one. The time analyzer is started by a pulse from an electromagnetic pick-up. The position of this pick-up should be adjusted so that the analyzer is started exactly when the neutrons pass the centre of the chopper (compare the theory of section III). To perform this calibration as exactly

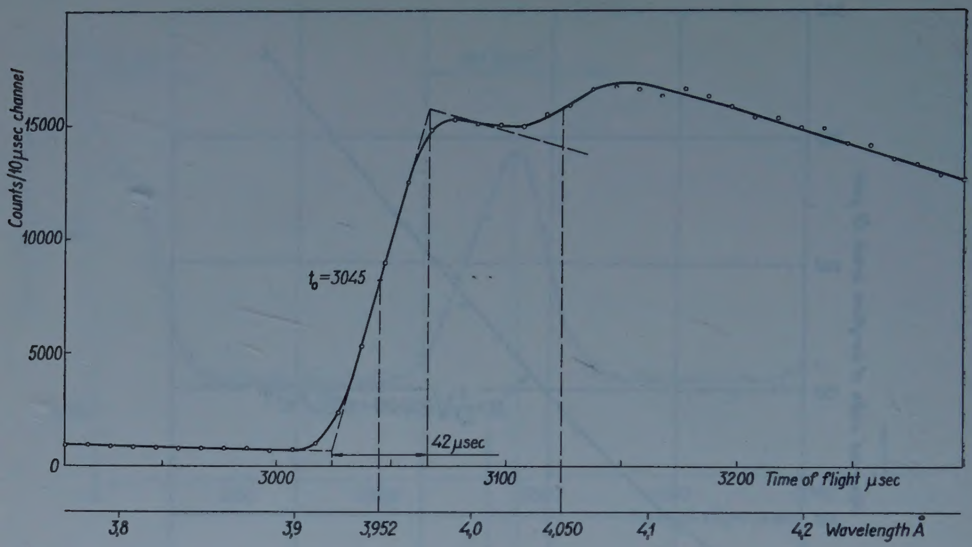

Fig. 11. Calibration of the time scale. Observed beryllium filtered spectrum after adjustment of pick-up position.

as possible, the highest possible resolution is desired. To this end the thick BF_3 detector was replaced by a thin, circular, boron-loaded ZnS scintillation detector of thickness $\leqslant 1$ mm, and of a diameter of 12.5 cm. Using this detector, which should give a calculated resolution half-width of 40 μsec at 4 Å at a chopper speed of 12,700 r.p.m. and a channel width of 10 μsec, the beryllium filtered reactor neutron spectrum was again recorded. The flight path from the chopper centre to the detector was 305.1 cm. As earlier discussed (ref. 3 appendix) the mid-point of the observed beryllium break very nearly defines the time-of-flight of the 3.952 Å neutrons. The position of the pick-up was adjusted until agreement between calculated and measured flight time was established to within about 2 μsec. The calibration was thus performed to an accuracy better than 0.1 %. The final result after the adjustment of the pick-up is shown in figure 11. As will be seen from this figure, the observed resolution width of 42 μsec agrees well with the calculated 40 μsec. Further it is obvious that this higher resolution is necessary for an accurate calibration because of the disturbing aluminium cut-off at 4.05 Å caused by constructional material in the reactor tank, beam ports and in the chopper itself. If a resolution of 80–100 μsec had been used, this aluminium cut-off would have remained unresolved, which means that a too long time-of-flight would have been associated with the 3.952 Å neutrons. In reality, a wave length between 3.95 and 4.05 Å would then have been used as the calibration wave length, and the adjustment of the pick-up position would have been in error.

Unfortunately the time scale is in this way calibrated just at one speed of rotation. As may be seen from figure 7, the position of the beryllium break apparently moves towards longer flight times when the chopper speed is reduced. This is an effect dependent upon the electrical inertia of the electromagnetic pick-up. Calculation shows that this time error should vary approximately as $1/\omega$ which agrees well with experiments. A faster triggering device such as a beam of light through the chopper observed by a photomultiplier would not give such a speed dependent start of the analyzer. The electromagnetic pick-up, however, is a simple device of a robust mechanical construction and gives steady, realiable pulses, facts which counteract the disadvantages just discussed.

To sum up, the chopper time-of-flight spectrometer is a flexible instrument to be used in the thermal and subthermal neutron energy regions. Its operation is understood in great detail, and with interchangeable choppers it may be used to solve any spectrometer problem in the energy region under consideration.

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